Circular Motion

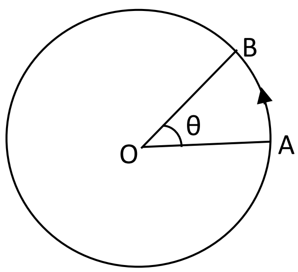
Circular motion:

Motion of a body in circular orbit is called circular motion. For example; motion of planets and satellites in their orbits, motion of electrons around the nucleus of an atom, motion of different hands of watch etc.

Terms used in circular motion:

Angular displacement:

The angle swept out by the radius vector of the particle at the centre of circular orbit in given interval of time is called angular displacement. Generally it is denoted by θ and its SI unit is radian. It is a vector quantity. Its direction is along the axis of rotation perpendicular to the plane of motion, given by right hand rule. So, it is an axial vector.



If a body moves from point A to B in time interval t, then θ represents the angular displacement of time interval t.

Angular velocity:

The rate of change of angular displacement with time is called angular velocity. It is denoted by ω. Its SI unit is radian per second (rad/sec). It is also an axial vector directed along the axis of rotation according to the right-hand rule.

If θ be the angular displacement of a body in time interval t, then average angular velocity is given by

$$\boldsymbol{\omega} = \frac{\boldsymbol{\theta}}{\boldsymbol{t}}$$

Alternatively, if θ_1 and θ_2 be the angular displacements of a body in time interval t_1 and t_2 from initial position, the average angular velocity between the time t_1 to t_2 is given by

$$\omega = \frac{\theta_2-\theta_1}{t_2-t_1} = \frac{\Delta\theta}{\Delta t} \quad \text{(where, } \Delta\theta = \theta_2 - \theta_1 \text{ \& } \Delta t = t_2 - t_1 \text{)}$$

The instantaneous angular velocity ω is given by the limiting value of average acceleration.

$$\text{i.e.,} \quad \omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$$

$$\Rightarrow \quad \boldsymbol{\omega} = \frac{\mathbf{d}\boldsymbol{\theta}}{\mathbf{d}\boldsymbol{t}}$$

i.e., the derivative of angular displacement with respect to time gives the instantaneous angular velocity.

For uniform circular motion, the average and instantaneous values of angular velocity are same.

Angular acceleration:

The rate of change of angular velocity is called angular acceleration. It is also an axial vector having a unit rad/s^2. It is denoted by α.

If the angular velocity of a body changes from initial value ω_o to final value ω in time interval t, then average angular acceleration is given by

$$\boldsymbol{\alpha} = \frac{\boldsymbol{\omega - \omega_o}}{\boldsymbol{t}}$$

Alternatively, if ω_1 and ω_2 are the angular velocities of a body in time t_1 and t_2 , then average angular velocity between the time t_1 to t_2 is given by

$$\alpha = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}$$

The instantaneous angular acceleration is given by the limiting value of average angular velocity.

i.e., $$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t}$$

$$\Rightarrow \quad \boldsymbol{\alpha} = \frac{\mathbf{d}\boldsymbol{\omega}}{\mathbf{d}\boldsymbol{t}}$$

i.e., the derivative of angular velocity with respect to time gives the instantaneous angular acceleration.

Time period:

Time period of a particle moving in a circular path is defined as the time taken by the particle to complete one revolution. It is denoted by T. Its SI unit is second.

$$\because \ \boldsymbol{\omega} = \frac{\boldsymbol{\theta}}{\boldsymbol{t}} \quad \text{and for t = T, } \theta = 2\pi,$$

$$\omega = \frac{2\pi}{T}$$

$$\therefore \ \boxed{\mathbf{T} = \frac{\boldsymbol{2\pi}}{\boldsymbol{\omega}}}$$

Frequency:

The frequency of a particle moving in a circular path is defined as the number of revolutions made by the particle in one second. It is denoted by f.

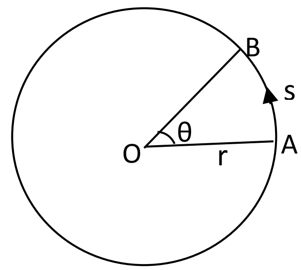
It is given by the relation,

$$\boldsymbol{f} = \frac{\mathbf{1}}{\boldsymbol{T}} \quad \text{(Its units are revolution/sec or cycle/sec or Hertz)}$$

$$\because \ \omega = \frac{2\pi}{T} = 2\pi \times \frac{1}{T}$$

$$\therefore \ \boxed{\boldsymbol{\omega = 2\pi f}}$$

Relation between linear velocity and angular velocity:



Consider a body moving in a circular path having centre at O and radius r. Let θ be the be the angular displacement of the body in time interval t and corresponding arc length is 's'.

From geometry, $\theta = \frac{s}{r}$ ($\because$ angle = $\frac{arc}{radius}$)

or, $s = r\theta$

Differentiating both sides with respect to t we get,

$$\frac{ds}{dt} = \frac{d(r\theta)}{dt} = r\frac{d\theta}{dt}$$

$\therefore$ $v = r\omega$

or, $\boxed{\mathbf{v = \omega r}}$

In vector form,

$$\vec{v} = \vec{\omega} \times \vec{r}$$

The direction of linear velocity at any point is given by the direction of tangent drawn at that point.

Differentiating $v = \omega r$ with respect to time, we get

$$\frac{dv}{dt} = \frac{d(\boldsymbol{\omega}\mathbf{r})}{dt} = r\frac{d\boldsymbol{\omega}}{dt} \quad (\because \text{ r is constant})$$

$\therefore$ $a = r\alpha$ (where $\alpha = \frac{d\boldsymbol{\omega}}{dt}$ is angular acceleration)

i.e., $\boxed{\mathbf{a = \alpha r}}$

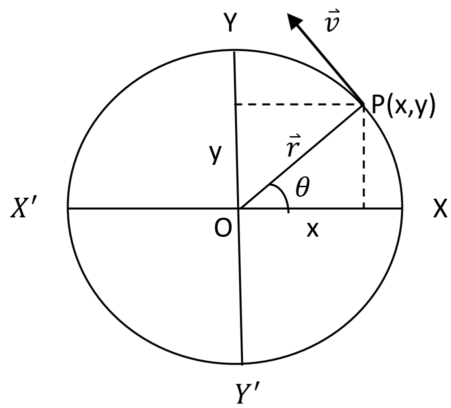
This is the relation between linear acceleration and angular acceleration.

Centripetal acceleration:

When a body moves with a uniform speed in a circular path; its velocity changes at every point due to change in direction. If there is change in velocity, there must be an acceleration. Such type of acceleration is called centripetal acceleration.

The acceleration of a body moving with a uniform speed in a circular orbit is called centripetal acceleration.

Expression for centripetal acceleration:



Consider a particle moving in a circular orbit in the XY plane with centre at O and radius r with uniform speed v. Let P (x,y) be the position of the particle at any time t, and the corresponding angular displacement is θ. The angular velocity of the particle is given by

$$\omega = \frac{\theta}{t}$$

$$\therefore \quad \theta = \omega t \quad \ldots\ldots\ldots\ldots\ldots\ldots\text{(i)}$$

The position vector of the particle at time t can be expressed as,

$$\vec{r} = x\,\hat{\imath} + y\,\hat{\jmath}$$

or, $\vec{r} = r\cos\theta\,\hat{\imath} + r\sin\theta\,\hat{\jmath} = r\,(\cos\omega t\,\hat{\imath} + \sin\omega t\,\hat{\jmath}) \quad \ldots\ldots\ldots\ldots\text{(ii)}$

Now, the velocity of the particle is given by,

$$\vec{v} = \frac{d\vec{r}}{dt} = r\omega\,(-\sin\omega t\,\hat{\imath} + \cos\omega t\,\hat{\jmath})$$

Also the acceleration of the particle is given by

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d[\,r\omega\,(-\sin\omega t\,\hat{\imath} + \cos\omega t\,\hat{\jmath})]}{dt} = -r\omega^2\,(\cos\omega t\,\hat{\imath} + \sin\omega t\,\hat{\jmath})$$

or, $\vec{a} = -\,\omega^2\,r\,(\cos\omega t\,\hat{\imath} + \sin\omega t\,\hat{\jmath})$

or, $\vec{a} = -\,\omega^2\vec{r} \quad \ldots\ldots\ldots\ldots\text{(iii)}$

This is the expression for centripetal acceleration. Negative sign shows that the direction of centripetal acceleration is opposite to the position vector $\vec{r}$. That is towards the centre of the circle.

In magnitude,

$$a = \omega^2 r$$

Also in terms of linear velocity, $a = \frac{v^2}{r}$ $\quad (\because v = \omega r)$

Alternative method for centripetal acceleration:

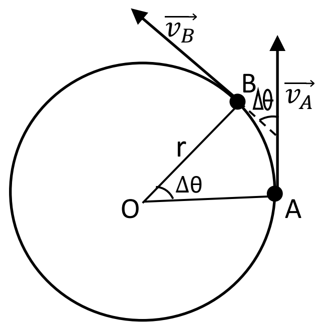


Fig : *i*

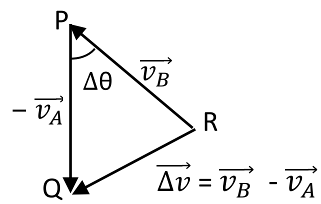


Fig : *ii*

Consider a body moving in a circular orbit having centre at O and radius r with uniform speed v. Let $\overrightarrow{v_A}$ and $\overrightarrow{v_B}$ are the velocities of the body at points A and B and angle subtended by the arc AB at the centre of the circle is $\Delta\theta$ as shown in fig i. To find the change in velocity in going from A to B, the velocities $-\overrightarrow{v_A}$ and $\overrightarrow{v_B}$ are represented by vectors $\overrightarrow{PQ}$ and $\overrightarrow{RP}$ as shown in fig ii. The resultant of the vectors $\overrightarrow{RP}$ and $\overrightarrow{PQ}$ is represented by the vector $\overrightarrow{RQ}$ which represents the change in velocity $\overrightarrow{v_B} - \overrightarrow{v_A} = \overrightarrow{\Delta v}$. Let Δt be the time taken by the body in going from A to B. For very small interval of time , i.e., for $\Delta t \rightarrow 0$, the points A and B are very close to each other and the angle $\Delta\theta$ is also small. For $\Delta\theta \rightarrow 0$, the change velocity $\overrightarrow{\Delta v}$ is perpendicular to both $\overrightarrow{v_A}$ and $\overrightarrow{v_B}$ and is towards the centre of the circle.

The magnitude of acceleration is given by

$$a = \frac{change\ in\ velocity}{time\ taken} = \frac{\Delta v}{\Delta t}$$

For small value of $\Delta\theta$,

$$\Delta\theta = \frac{RQ}{RP} = \frac{\Delta v}{v} \quad [\because |\overrightarrow{RP}| = |\overrightarrow{PQ}| = v]$$

i.e., $\Delta v = v\,\Delta\theta$

$$\therefore \quad a = \frac{v\,\Delta\theta}{\Delta t}$$

For $\Delta t \rightarrow 0$,

$$a = \Delta t \rightarrow 0 \ \frac{v\,\Delta\theta}{\Delta t} = v\frac{d\theta}{dt} \quad [\because \text{ v is constant }]$$

$$\therefore \quad a = v\omega = v \times \frac{v}{r}$$

$$\Rightarrow \quad \boxed{a = \frac{v^2}{r}}$$

This is the expression for centripetal acceleration. As the direction of change in velocity is towards the centre of the circle, the direction of centripetal acceleration is also towards the centre of the circle.

Also in terms of angular velocity,

$$\boxed{\mathbf{a} = \boldsymbol{\omega}^2\mathbf{r}}$$

Centripetal force:

When a body moves with a uniform speed in a circular path; its velocity changes at every point due to change in direction. If there is change in velocity, the body has acceleration and this acceleration is towards the centre of the circle. A constant force towards the centre of the circle is required to maintain this acceleration which is called centripetal force.

The force required for a body to move with uniform speed in a circular path is called centripetal force. It is given by the relation,

F = mass × centripetal acceleration

i.e., $$\boxed{F = \frac{mv^2}{r}}$$

Also in terms of angular velocity,

$$\boxed{F = m\omega^2 r}$$

The direction of centripetal force is same as that of centripetal acceleration. i.e. towards the centre of the circle. Without centripetal force nobody can move in circular orbit.

Examples:

- **i.** The centripetal force for the planets to move around the sun is provided by the gravitational force between the sun and the planet.
- **ii.** The centripetal force for the electrons to move around the nucleus of an atom is provided by the electrostatic force between the nucleus and the electrons.

Centrifugal force:

The outward force experienced by a body moving in a circular path is called centrifugal force. The magnitude of centrifugal force is same as that of centripetal force but direction is away from the centre of the circle.

i.e., centrifugal force, $F = \frac{mv^2}{r} = m\omega^2 r$

The centrifugal force is not a real force but a fictitious force due to inertial property of the body. The centrifugal force on the heavy particle is greater than that on the light particles. When a mixture containing light particles and heavy particles; is set into circular motion then the heavy particles move away from the axis of rotation and the light particles stay close to the axis. In this way the heavy particles and the light particles of the mixture are separated. This is the principle of centrifuge.

Applications of centrifuge:

a. Cream is separated from milk in cream separator.
b. Wet clothes are dried in drying machine.
c. Centrifuges are used in sugar industry to separate sugar crystal from molasses.
d. Centrifuges are used to separate honey from wax.

Applications of centripetal forces

1. **Motion of a vehicle in a level curved path:**

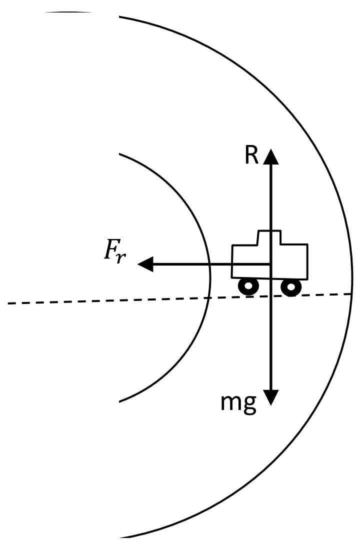


Fig : Motion of vehicle in a level curved path

Consider a vehicle of mass m moves in a level curved path of radius r with speed v. The frictional force between the tyre and the road acts towards the centre of the circle and provides necessary centripetal force.

i.e., $F_c = F_r$

or, $\frac{mv^2}{r} = \mu R$ (where μ is the coefficient friction between the tyre and the road)

or, $\frac{mv^2}{r} = \mu$ mg

$\therefore$ $\mathbf{v = \sqrt{\mu\, rg}}$

This is the maximum velocity with which the vehicle can take a turn in a level curved path of radius r.

2. Banking of road :

The phenomenon of raising of outer edge of the road over the inner edge in a curved path is called banking of road. The angle by which outer edge is raised over the inner edge is called banking angle or angle of banking. Roads are banked in curved path to make the centripetal force independent to the force of friction.

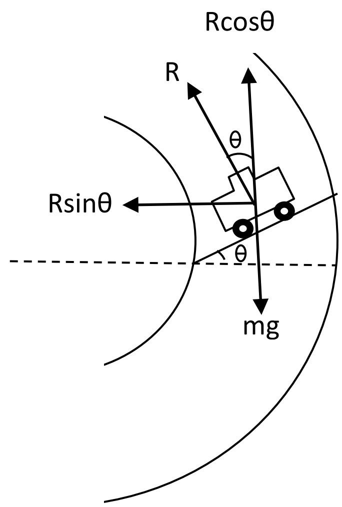


Fig : Banking of road

Consider a vehicle moving with speed v in a curved path of radius r having banking angle θ. In banked road, the normal reaction R of the ground to the vehicle makes same angle θ with the vertical. The normal reaction R is resolved into two components; the vertical component R$\cos\theta$ balances the weight mg of the vehicle and horizontal component R$\sin\theta$ provides necessary centripetal force.

i.e., $R\cos\theta = mg$ ....(i)

And, $R\sin\theta = \frac{mv^2}{r}$ ....(ii)

Dividing eq^n (ii) by (i), we get

$$\tan\theta = \frac{v^2}{rg}$$

or, $\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$ ....(iii)

This is the expression for the angle by which the road should be banked in a curved path of radius r to move safely with speed v.

Also the safe limit of velocity can be expressed as,

$v = \sqrt{rg\tan\theta}$ ....(iv)

3. **Bending of cyclist:**

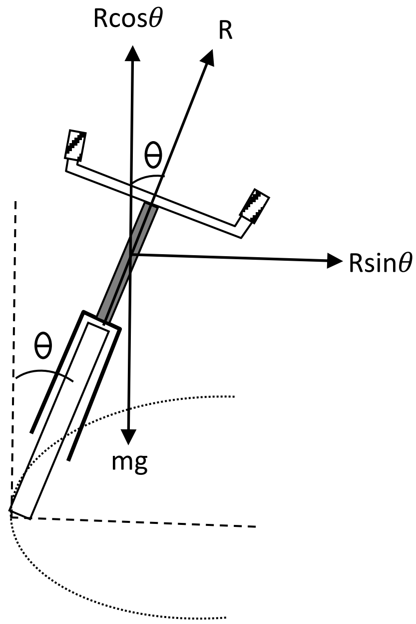


Fig : Bending of cyclist

Consider a cyclist taking a turn in a curved path of radius r with speed v. While taking a turn, the cyclist inclines himself with vertical. Let θ be the angle by which he inclines himself with vertical. While he inclines, the normal reaction R of the ground also makes angle θ with vertical. The normal reaction R is resolved into two components; the vertical component R$\cos\theta$ balances the weight mg of the cyclist and horizontal component R$\sin\theta$ provides necessary centripetal force.

i.e., $R\cos\theta = mg$ ....(i)

And, $R\sin\theta = \frac{mv^2}{r}$ ....(ii)

Dividing eq^n (ii) by (i) ,we get

$$\tan\theta = \frac{v^2}{rg}$$

or, $\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$ ....(iii)

This is the expression for the angle by which the cyclist should incline himself with vertical while taking a turn in a curved path of radius r with speed v.

4. Motion in a horizontal circle - Conical Pendulum:

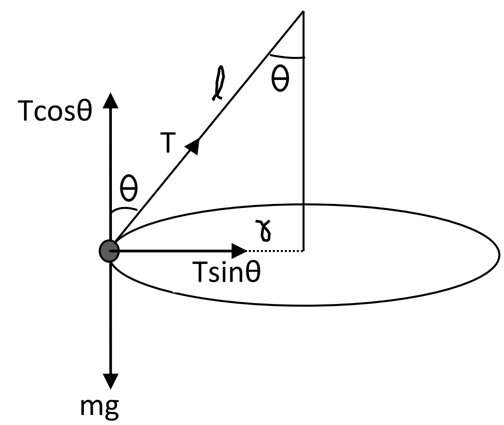


Fig : Conical Pendulum

Consider a body of mass m is tied to a string of length l and whirled round a horizontal circle of radius r with uniform speed v. Let θ be the angle made by the string with vertical. When the mass moves in a horizontal circle, the motion of the string describes a cone. So it is also called conical pendulum. When the mass moves in a horizontal circle, the tension 'T' in the string acting towards the point of suspension can be resolved into two components. The vertical component Tcosθ balances the weight mg of the mass and the horizontal component Tsinθ provides necessary centripetal force.

i.e., $T\cos\theta = mg$ ....(i)

And, $T\sin\theta = \frac{mv^2}{r}$ ....(ii)

Dividing eq^n (ii) by (i), we get

$$\tan\theta = \frac{v^2}{rg}$$

or, $v = \sqrt{rg\tan\theta}$

or, $\omega r = \sqrt{rg\tan\theta}$ (where ω is angular velocity of the mass)

or, $\omega = \sqrt{\frac{g\tan\theta}{r}}$

Now the time period of conical pendulum is given by

$$t = \frac{2\pi}{\omega}$$

or, $t = \frac{2\pi}{\sqrt{\frac{g\tan\theta}{r}}} = 2\pi\sqrt{\frac{r}{g\tan\theta}}$

From figure, $\sin\theta = \frac{r}{l} \Rightarrow r = l\sin\theta$

$$\therefore \quad t = 2\pi\sqrt{\frac{l \sin\theta}{g \tan\theta}} = 2\pi\sqrt{\frac{l \sin\theta \times \cos\theta}{g \sin\theta}}$$

or,

$$\boxed{t = 2\pi\sqrt{\frac{l \cos\theta}{g}}}$$

This is the expression for the time period of conical pendulum.

5. **Motion in a Vertical circle:**

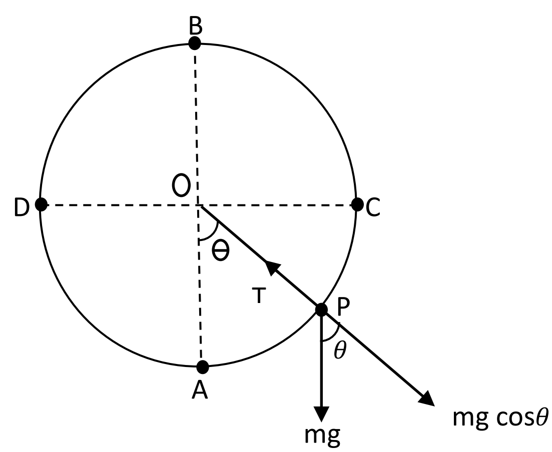


Fig : Motion in a vertical circle

Consider a body of mass m is tied to one end of a string and whirled round a vertical circle of radius r with speed v. When the body moves in the vertical circle, the tension T in the string always acts towards the centre of the circle. Let at any instant, the mass is at point P and the angle made by the string with vertical is θ. When the mass is at point P, the component of weight mg of the mass acting opposite to the tension is mg cosθ. So, the resultant force acting towards the centre of the circle is T – mg cosθ, which provides the necessary centripetal force for the mass to move in the circular path.

i.e., $\quad T - mg\cos\theta = \frac{mv^2}{r}$

or, $\quad T = \frac{mv^2}{r} + mg\cos\theta$ ....(i)

This is the expression for the tension in the string; when a mass tied to a string is rotated in a vertical circle.

Cases:

i. When the mass is at lowest point of vertical circle (at point A), $\theta = 0^o$

$\therefore \quad T = \frac{mv^2}{r} + mg$

ii. When the mass is at highest point of vertical circle (at point B), $\theta = 180^o$

$$\therefore \quad T = \frac{mv^2}{r} - mg$$

iii. When the string is horizontal (mass at point C or D), $\theta = 90^o$ or 270^o

$$\therefore \quad T = \frac{mv^2}{r}$$

Thus, the tension in the string is maximum when the mass is at lowest point of vertical circle and minimum when the mass is at highest point of vertical circle.

i.e., $T_{max} = \frac{mv^2}{r} + mg$

And, $T_{min} = \frac{mv^2}{r} - mg$

***Note** : When the string is rotated in a horizontal plane with a uniform speed (motion in horizontal circle), the tension in the string remains constant (i.e., $T = \frac{mv^2}{r}$ or $T = m\omega^2 r$).*

Minimum velocity of a mass to move in a vertical circle:

Let v_1 and v_2 be the minimum velocity of a mass at the highest and lowest point of vertical circle. The tension in the string is minimum when the mass is at highest point of vertical circle.

i.e., $T_{min} = \frac{mv^2}{r} - mg$

If $T_{min} = 0$, the weight mg of the mass provides necessary centripetal force and the mass moves with minimum velocity v_1 at the highest point.

i.e., $0 = \frac{mv_1^2}{r} - mg$

or, $\frac{mv_1^2}{r} = mg$

$\Rightarrow$ $\mathbf{v_1 = \sqrt{rg}}$ **......................(i)**

This is the expression for minimum velocity of a mass at the highest point of vertical circle called critical velocity. If the velocity at highest point is less than the critical velocity, the mass does not move in vertical circle but falls vertically from the highest point.

When the mass moves from highest point to lowest point in the vertical circle, the velocity goes on increasing and becomes v_2 at the lowest point.

From conservation of energy,

Gain in KE = Loss in PE

i.e., $\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = mg\times 2r$

or. $v_2^2 - v_1^2 = 4rg$

or. $v_2^2 - rg = 4rg$ $[\because v_1 = \sqrt{rg}\,]$

$\therefore$ $\mathbf{v_2 = \sqrt{5rg}}$ **........................(ii)**

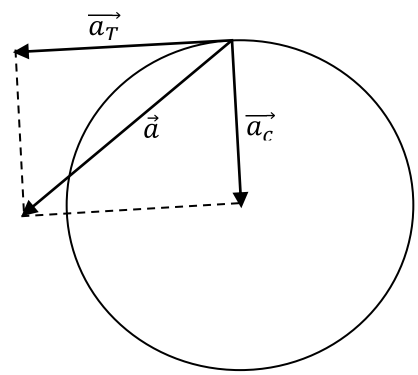
Similarly, the minimum velocity of mass when the string becomes horizontal is given by

v_3 = $\sqrt{3rg}$ ....(iii)

Conceptual questions:

1. Is it necessary that the acceleration in circular motion is always towards the centre of the circle? Explain.

Ans : There are two types of acceleration in circular motion. One is centripetal acceleration or radial acceleration ($\overrightarrow{a_c}$), which acts towards the centre of the circle and keeps the body in circular path. Another is tangential acceleration ($\overrightarrow{a_T}$) which changes the speed of the body. So, the acceleration is circular motion is given by the resultant of these two accelerations.



i.e., $\vec{a} = \overrightarrow{a_c} + \overrightarrow{a_T}$

Cases :

i. If $\overrightarrow{a_T} = 0$, $\overrightarrow{a_c} \neq 0$, then $\vec{a} = \overrightarrow{a_c}$

i.e., the acceleration is towards to centre of the circle and the motion is uniform circular motion.

ii. If $\overrightarrow{a_T} \neq 0$, $\overrightarrow{a_c} \neq 0$, then $\vec{a} = \overrightarrow{a_c} + \overrightarrow{a_T}$
with magnitude ; $a = \sqrt{a_c^2 + a_T^2}$

i.e., the acceleration is not towards to centre of the circle and the motion is non uniform circular motion.

iii. If $\overrightarrow{a_T} \neq 0$, $\overrightarrow{a_c} = 0$, then $\vec{a} = \overrightarrow{a_T}$ and the motion is linear but not circular

Thus, in case of uniform circular motion, the acceleration is towards the centre of the circle and in case of non-uniform circular motion it is not towards the centre of the circle.

2. Does centripetal force do a work? Explain.

Ans : Work done by a force is given by

$$W = FS\cos\theta$$

In case of centripetal force, the force and displacement are perpendicular. i.e. $\theta = 90^o$

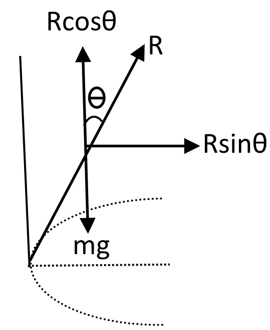
$$\therefore \quad W = FS\cos 90^o = 0$$

Thus, the centripetal force does not do any work.

3. Why is it more difficult to revolve a stone by tying it to a longer string than a shorter string?
Ans : To move a body in circular path , a force towards the centre called centripetal force is required which is given by, $F = m\omega^2 r$.
Thus to revolve a stone in longer string, a larger centripetal force is required. So, it is more difficult to revolve a stone by tying it to a longer string than a shorter string.
4. The positively charged nucleus of an atom attracts the electrons in the orbit. Why do the electrons not collapse into the nucleus?

Ans: The electrostatic force between the nucleus and electrons is used to provide necessary centripetal force for the electrons to move in circular orbit. In case of centripetal force, the force and displacement are perpendicular to each other and the force cannot do any work to pull the electrons towards the centre ($W = FS \cos 90^o = 0$). So, the electrons do not collapse into the nucleus.

5. Why does a cyclist incline himself with vertical while taking turn in a curved path?



Ans : When he inclines himself with vertical, the normal reaction of the ground makes some angle with vertical. The vertical component of normal reaction balances the weight and horizontal component provides necessary centripetal force to move in the curved path as shown in fig.

Numerical Probems:

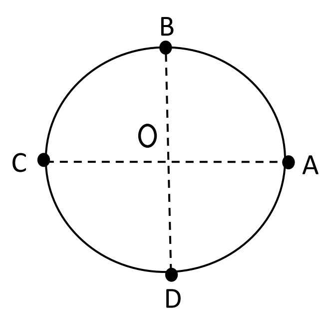
1. A bob of mass 200 gm is whirled in a horizontal circle of radius 50 cm by a string inclined at 30^o to the vertical. Calculate the tension in the string and the speed of the bob in the horizontal circle. **(Ans: 2.3 N, 1.7 m/s)**
2. An object of mass 0.5 kg is rotated in a horizontal circle by a string 1m in length. The maximum tension in the string before it breaks is 50 N. What is the greatest number of revolution per second of the object? **(Ans: 1.6 rev/s)**
3. A mass of 0.2 kg is rotated by a string at constant speed in a vertical circle of radius 1 m. If the minimum tension in the string is 3 N, calculate the magnitude of speed and the maximum tension in the string. **(Ans: 5 m/s, 7 N)**
4. At what angle should a circular road be banked so that a car running at 50 km/hr is safe to go round the circular turn of 200 m radius? **(Ans: 5.5^o)**
5. A certain string breaks when a weight of 25 N acts on it. A mass of 500 gm is attached to one end of the string of 1 m long and is rotated in a horizontal circle. Find the greatest number of revolutions per minute which can be made without breaking the string. **(Ans: 67.5 rpm)**
6. A bob of mass 1 kg is attached to the lower end of the string 1 m long whose upper end is fixed. The mass is made to rotate in a horizontal circle of radius 60 cm. If the circular speed of the mass is constant, find the tension in string and the period of motion. **(Ans: 12.5 N, 1.77 s)**
7. A coin is placed on a disc rotating with speed of $33\frac{1}{3}$ rev/min provided that the coin is not more than 10 cm from the axis. Calculate the coefficient of static friction between the coin and the disc. **(Ans: 0.12)**

8. A body of mass 200 gm is attached to a fixed point by a string of length 0.5m. It is held with the string horizontal and then let fall. Find the velocity when the string becomes vertical and the tension in the string then. **(Ans: 3.16 m/s, 6 N)**
9. A stone of mass 100 gm is tied to one end of a string of length 1m and rotated in a vertical plane. The speed is gradually increasing until the string breaks. If the string can bear a tension up to 75 N, find the speed at which the string breaks. **(Ans: 27.2m/s)**
10. A 5 kg box rests on the floor of a merry go- round, 2 m from the centre. If the coefficient of sliding friction is 0.25, find the angular velocity at which the box will just begin to slide. **(Ans: 1.1 rad/s)**
11. A cyclist rides at constant velocity of 36 km/hr along a circle of radius 20 m. At what angle to the vertical should he incline his bicycle? **(Ans: 26.6^o)**
12. A small coin is placed on a flat horizontal turn table. The turn table is rotating at 60 revolutions/min. what is the coefficient of static friction between the coin and the turn table if the coin is observed to slide off the turn table when it is greater than 12 cm from the centre of turn table? **(Ans: 0.47)**
13. A body of mass 10 kg is weighed on a spring balance in the carriage of a train which is moving along a curve path of radius 400 m and at uniform speed of 20 m/s. What will be the reading of the spring balance? **(Ans : 100.5 N)**
14. A cyclist speeding at 18 km/h on a level road takes a sharp circular turn of radius 3 m without reducing the speed and without bending towards the centre of the circular path. The coefficient of static friction between the tyres and the road is 0.1. Will the cyclist slip while taking the turn? **(Ans: The cyclist will slip)**
15. A coin is placed on a horizontal turntable rotating at $33\frac{1}{3}$ rpm. The coin revolves with the table without slipping provided that the coin is not more than 10 cm away from the axis. How far from the axis can the coin be placed so that it revolves with the table without slipping if the turntable rotates at 45 rpm? **(Ans: 5.5 cm)**
16. A car of mass 1200 kg is moving on a circular track of radius 50 m with a speed of 10 m/s**.** Calculate the centripetal force acting on the car. **(Ans : 2400N)**
17. A stunt rider on a motorcycle is attempting a vertical loop of radius 5 m. What is the minimum speed the rider must have at the highest point to stay in contact with the loop? (Take g = 9.8 m/s^2). **(Ans: 7 m/s)**
18. A curved road is designed for vehicles to safely travel at a speed of 72 km/h without relying on friction. The radius of curvature of the road is 60 m, calculate the angle of banking required for safe turning. (Take g = 9.8 m/s^2) **(Ans: 34.2^o)**

<u>Multiple choice questions:</u>

1. A bucket tied at the end of 1.6 m long string is whirled in a vertical circle with constant speed. The minimum speed at which water does not spill when the bucket is at highest position is :
   a) 4 m/s b) 6.25 m/s c) 2 m/s d) 16 m/s
2. A particle covers equal distance around a circular path in equal interval of time. The quantity remains constant is:
   a) Displacement b) velocity **c) speed** d) acceleration

3. The angular speed of hour hand of clock is:
   a) 2π rad/sec b) 2π rad/min c) 2π rad/day **d) $\frac{\pi}{6}$ rad/hr**
4. A car is moving with speed 30 m/s on a circular path of radius 500 m. Its speed is increasing at the rate of 2 m/s^2. What is the acceleration of the car?
   a) 2 m/s^2 **b) 2.7 m/s^2** c) 1.8 m/s^2 d) 9.8 m/s^2
5. What is the ratio of angular speed of minute hand and second hand of a clock?
   a) 1 : 12 b) 12 : 1 **c) 1 : 60** d) 60 : 1
6. A particle is moving along a circular path. The angular velocity, linear velocity, angular acceleration and centripetal acceleration are denoted by $\vec{\omega}$, $\vec{v}$, $\vec{\alpha}$ and $\overrightarrow{a_c}$, which of the following statement is not correct?
   a) $\vec{\omega} \perp \overrightarrow{a_c}$ **b) $\vec{\omega} \perp \vec{\alpha}$** c) $\vec{\omega} \perp \vec{v}$ d) $\vec{v} \perp \overrightarrow{a_c}$
7. When a body moves with a constant speed along a circle:
   a) Its velocity remains constant b) No force acts on it
   c) No work is done on it d) No acceleration is produced in it
8. A string can withstand a tension of 25 N, What is the greatest speed at which a body of mass 1kg can be whirled in a horizontal circle using 1m length of the string?
   a) 2.5 m/s **b) 5 m/s** c) 7.5 m/s d) 10m/s
9. A body crosses the topmost point of a vertical circle with critical speed. What will be its acceleration when the string is horizontal?
   a) g b) 2g **c) 3g** d) 6g
10. An object is tied to a string and rotated in a vertical circle of radius r. Constant speed v is maintained along the trajectory. If T_{max}/ T_{min} = 2, then v^2/rg is :
   a) 1 b) 2 **c) 3** d) 4
11. Figure shows a body of mass m moving with a uniform speed v along a circle of radius r. The change in speed in going from C to D is :



   a) v b) v/2 c) v/$\sqrt{2}$ **d) Zero**

12. In question 11, change in velocity in going from A to B is :
   a) v$\sqrt{2}$ b) v c) v/$\sqrt{2}$ d) Zero